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MHD Peristaltic Flow of a Couple Stress Fluids Permeated with Suspended Particles

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Abstract— The MHD peristaltic flow of a couple stress fluid permeated with suspended particles through a two dimensional flexible channel under long wave length and low Reynolds approximation is studied. A analytical method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity in fluid phase and particle phase. The expression for velocity profile and pressure gradient and the volumetric flow rate in the wave frame is obtained. The graphical results obtained for velocity profile and pressure gradient. It is observe that velocity profile decreases with increase in Hartmann number M and couple stress parameter S. The pressure gradient has an opposite behavior compared with velocity profile for Hartmann number M.

ndex Terms— Peristaltic flow, Couple Stress, MHD, Suspended Particles, Velocity Profile, Pressure Gradient

1 Introduction

The movement of body fluids travelling through tubular organs continuously by contraction and relaxation due to change in pressure along the walls is called peristalsis. Peristaltic movement takes place while swallowing food through the oesophagus to the stomach, transport of lymph on lymphatic vessels, vasomotion of small blood vessels tube, capillaries vennules and arteries, transport of chyme through the small intestine, the feces movement in the large intestine, the passage of urine from the kidneys through the ureter to the urinary bladder, the flow of semen in the male reproductive tract, and the movement of ovum in the female fallopian tube. Peristaltic phenomenon is used in biomedical instruments like dialysis machines, heart lung machines, artificial heart and ortho machines and transport of toxic material waste inside the sanitary ducts, peristaltic concept is used in nuclear industry to avoid contamination of the outside environment.

For the recent contribution, we refer the reader to [1—20] and the references cited therein. The study of peristaltic transport in experimental and analytical situations has recently become the object of scientific research, since the first investigation by T.W.Latham [3]. J.C.Burns et.al [2] has first considered the low Reynolds number in his article of peristaltic motion. A very important investigation is made by Shapiro et.al [6]. He only initiated the concept of long wavelength and low Reynolds number approximation. Brown and Hung [1] had set the Reynolds number as finite. Moreover they discussed two dimensional peristaltic flow in both computational and experimental situations in detail.

The theory of couple stress was first developed by Stokes 1966 [14] and represents the simplegeneralization of classical theory which allows for polar effects such as presence of couplestress and body couples. A few examples of such fluids consisting of rigid, randomlyoriented particles (red cells), suspended in a viscous medium, such as blood, lubricantscontaining small amount of polymer additive, electro-rheological fluids and synthetic fluids.Several authors Valanis and Sun (1969), Chaturani (1978), Chaturani and Rathod (1981),Srivastava (1986), Mekheimer (2002), Sobh (2008), Raghunatha Rao and Prasada Rao (2012) havestudied couple stresses in peristaltic flow [15, 8, 9, 13, 4, 12, 10].

The study of two-phase flows finds applications in many branches of Engineering,Environmental, Physical Sciences, etc. A few examples of such flows in diverse fields are theflow of dissolved micro molecules of fiber suspensions in paper making, flow of bloodthrough arteries, propulsion and combustion in rockets, dispersion and fall out of pollutants inair, erosion of material due to continuous impingement of suspended particles in air etc. It dusty fluid servesas a better model to describe blood as a binary system. Solid-particle motion in two-dimensional peristaltic flows has been discussed by Hung and Brown 1976 [1]. Dust velocity shear driven rotational waves andassociated vertices in a non-uniform dusty plasma has been investigated by M. H. Subba Reddy et al.(2012) [19],G. Rami Reddy et.al (2011) [20] studied unsteady flow of a dusty fluid between two oscillating platesunder varying constant pressure gradient, Ravi kumar et al.(2010) [17] discussed the peristalticflow of a dusty couple stress fluid in a flexible channel.

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In this present article MHD peristaltic flow of a couple stress fluid permeated with suspended particles through a two dimensional flexible channel under long wave length and low Reynolds approximation is studied. A analytical method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity in fluid phase and particle phase. The expression for velocity profile and pressure gradient and the volumetric flow rate in the wave frame is obtained. The graphical results obtained for velocity profile and pressure gradient for various parameter.

2 Formulation of the Problem

We consider a peristaltic flow of a couple stress-dusty fluids through two-dimensional channel bounded by flexible walls. The geometry of the flexible walls are represented by

$$y=\eta\left(X,t\right)=d+a\sin\frac{2\pi}{\lambda}\left(X-ct\right) \tag{1}$$

Where 'd' is the mean half width of the channel, 'a' is the amplitude of the peristaltic wave, 'c' is the wave velocity. λ is the wave length and t is the time.

The equations governing the two-dimensional flow of a couple stress fluid permeated withsuspended particles in fluid phase and particle phase are

$$\frac{\partial\overline{u}}{\partial\overline{x}}+\frac{\partial\overline{v}}{\partial\overline{y}}=0 \tag{2}$$

$$\rho\left(\frac{\partial\overline{u}}{\partial\overline{t}}+\overline{u}\frac{\partial\overline{u}}{\partial\overline{x}}+\overline{v}\frac{\partial\overline{u}}{\partial\overline{y}}\right)=-\frac{\partial\overline{p}}{\partial\overline{x}}+\mu\nabla^2\overline{u}+\eta^*\nabla^4\overline{u}+ KN(\overline{u}_p-\overline{u})-\sigma B_0^2\overline{u} \tag{3}$$

$$\rho\left(\frac{\partial\overline{v}}{\partial\overline{t}}+\overline{u}\frac{\partial\overline{v}}{\partial\overline{x}}+\overline{v}\frac{\partial\overline{v}}{\partial\overline{y}}\right)=-\frac{\partial\overline{p}}{\partial\overline{y}}+\mu\nabla^2\overline{v}+ \eta^*\nabla^4\overline{v}+KN(\overline{v}_p-\overline{v}) \tag{4}$$

For dust particles

$$\frac{\partial N}{\partial\overline{x}}+N\left(\frac{\partial\overline{u}_p}{\partial\overline{x}}+\frac{\partial\overline{v}_p}{\partial\overline{y}}\right)=0 \tag{5}$$

$$\left(\frac{\partial\overline{u}_p}{\partial\overline{t}}+\overline{u}_p\frac{\partial\overline{u}_p}{\partial\overline{x}}+\overline{v}_p\frac{\partial\overline{u}_p}{\partial\overline{y}}\right)=\frac{k}{m}(\overline{u}-\overline{u}_p) \tag{6}$$

$$\left(\frac{\partial\overline{v}_p}{\partial\overline{t}}+\overline{u}_p\frac{\partial\overline{v}_p}{\partial\overline{x}}+\overline{v}_p\frac{\partial\overline{v}_p}{\partial\overline{y}}\right)=\frac{k}{m}(\overline{v}-\overline{v}_p) \tag{7}$$

$$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right),\nabla^4=\nabla^2\nabla^2 \tag{8}$$

where $\overline{u},\overline{v}$ is the velocity of the fluid particles, $\overline{u}_p,\overline{v}_p$ is the velocity of the dust particles, $\overline{p}$ is the fluid pressure, σ is the electrical conductivity of the fluid, B_0 is the applied magnetic field, ϱ is the density of the fluid, υ is the kinematic coefficient of the viscosity of fluid, and $K=6\pi\mu r$, r being the particle radius, is the Stoke's drag coefficient for the dust particles(a constant), m is the mass of the solid particles, η is the coefficient of couple stress, N is the number density of the particle, μ is the coefficient of viscosity, k is the stokes resistance coefficient.

The corresponding boundary conditions are

$$\overline{u}=0 \quad at \quad \overline{y}=\pm\eta$$

$$\frac{\partial^2\overline{u}}{\partial\overline{y}^2}=0 \quad at \quad \overline{y}=\pm\eta \tag{9}$$

$$\overline{v}=0 \quad at \quad \overline{y}=0$$

Indroducing a wave frame (x,y) moving with velocity c away from the fixed frame (X,Y) by the transformation

$$\overline{x}=X-c\overline{t},\overline{y}=Y,\overline{u}=U-c,\overline{v}=V,\overline{p}=P(X,t)$$

Indroducing the following non dimensional quantities

$$x=\frac{\overline{x}}{\lambda},y=\frac{\overline{y}}{d},u=\frac{\overline{u}}{c},u_p=\frac{\overline{u}_p}{c},v=\frac{\overline{v}}{c},v_p=\frac{\overline{v}_p}{c},\eta=\frac{\overline{\eta}}{d}$$

$$,t=\frac{\upsilon\overline{t}}{\lambda},\psi=\frac{\overline{\psi}}{\upsilon},\phi=\frac{\overline{\phi}}{(Kd^2/m)}$$

these equations reduces to

$$y=\eta\left(x\right)=1+\varepsilon\sin 2\pi x \tag{10}$$

$$R\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+ S\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{R\alpha}{\tau}(u_p-u)-M^2u \tag{11}$$

$$R\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+ S\delta^2\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{R\alpha}{\tau}\delta^2(v_p-v) \tag{12}$$

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For dust particles

$$\delta\left(\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{1}{\tau}(u-u_p) \qquad (13)$$

$$\delta\left(\frac{\partial v_p}{\partial t}+v_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{1}{\tau}(v-v_p) \qquad (14)$$

Where $\varepsilon=\frac{a}{d}$ and $\delta=\frac{d}{\lambda}$ are geometric parameters, $R=\frac{cd}{\nu}$ is the Reynolds number , $S=\frac{\eta^*}{\mu d^2}$ is the couple stress parameter, $\alpha=\frac{Nm}{\rho}$ is the dust concentration parameter, $\tau=\frac{cm}{kd}$ is the relaxation time, $M^2=B_0 d\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number

The corresponding non dimensional boundary conditions are

$$u=-1 \quad at \quad y=\pm\eta$$
$$\frac{\partial^2 u}{\partial y^2}=0 \quad at \quad y=\pm\eta \qquad (15)$$
$$v=0 \quad at \quad y=0$$

3 Method of Solution

Using approximation of the long wavelength $(i.e\,\delta \ll 1)$ and neglecting the wave number and with approximation of the low Reynolds number $(i.e\,\mathrm{Re}\to 0)$, we get

$$S\frac{\partial^4 u}{\partial y^4}-\frac{\partial^2 u}{\partial y^2}-\frac{R\alpha}{\tau}(u_p-u)+M^2u=-\frac{\partial p}{\partial x} \qquad (16)$$

$$0=-\frac{\partial p}{\partial y} \qquad (17)$$

$$0=\frac{1}{\tau}(u-u_p) \qquad (18)$$

$$0=\frac{1}{\tau}(v-v_p) \qquad (19)$$

The corresponding non dimensional boundary conditions are

$$u=-1 \quad at \quad y=\pm\eta$$
$$\frac{\partial^2 u}{\partial y^2}=0 \quad at \quad y=\pm\eta \qquad (20)$$
$$v=0 \quad at \quad y=0$$

Solving the equations subject to boundary conditions we get

$$u=T_1\cosh\alpha y+T_2\cosh\beta y-\frac{dp/dx}{M^2} \qquad (21)$$

$$u_p=T_3\cosh\alpha y+T_4\cosh\beta y+T_5 \qquad (22)$$

$$v=v_p=-\frac{T_6\sinh\alpha y}{\alpha}-\frac{T_7\sinh\beta y}{\beta} \qquad (23)$$

Knowing the velocity, the volume flow rate q in a wave frame of reference is given by

$$q=\int_0^\eta u\,dy=\frac{T_1\sinh\alpha\eta}{\alpha}+\frac{T_2\sin\beta\eta}{\beta}-\frac{(dp/dx)\eta}{M^2} \qquad (24)$$

The pressure gradient is

$$\frac{dp}{dx}=\frac{M^2\left(q+\dfrac{\tanh\alpha\eta}{\alpha\left(1-\dfrac{\alpha^2}{\beta^2}\right)}+\dfrac{\tanh\beta\eta}{\beta\left(1-\dfrac{\beta^2}{\alpha^2}\right)}\right)}{\dfrac{\tanh\alpha\eta}{\alpha\left(1-\dfrac{\alpha^2}{\beta^2}\right)}+\dfrac{\tanh\beta\eta}{\beta\left(1-\dfrac{\beta^2}{\alpha^2}\right)}-\eta} \qquad (25)$$

where $\alpha^2=\frac{1+\sqrt{1-4SM}}{2S},\ \beta^2=\frac{1-\sqrt{1-4SM}}{2S}$

Where

$$T_1=\left(\frac{dp/dx}{M^2}-1\right)\frac{1}{\cosh\alpha\eta\left(1-\dfrac{\alpha^2}{\beta^2}\right)},$$

$$T_2=\left(\frac{dp/dx}{M^2}-1\right)\frac{1}{\cosh\beta\eta\left(1-\dfrac{\beta^2}{\alpha^2}\right)}$$

$$T_3=\frac{T_2S\tau\beta^4}{R\alpha_1}-\frac{T_2\beta^2\tau}{R\alpha_1}+T_2\left(\frac{R\alpha_1}{\tau}+M^2\right),$$

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$$T_4 = \frac{T_1 S \tau \alpha^4}{R\alpha_1} - \frac{T_1 \alpha^2 \tau}{R\alpha_1} + T_1\left(\frac{R\alpha_1}{\tau} + M^2\right)$$

$$T_5 = \frac{(dp/dx)\tau}{R\alpha_1} - \left(\frac{R\alpha_1}{\tau} + M^2\right)\frac{(dp/dx)}{M^2}.\frac{\tau}{R\alpha_1},$$

$$T_6 = \frac{\left(\dfrac{dp/dx}{M^2} - 1\right)\alpha \sinh \alpha\eta \left(2\pi\varepsilon \cos(2\pi x)\right)}{\cosh^2 \alpha\eta \left(1 - \dfrac{\alpha^2}{\beta^2}\right)}$$

$$T_7 = \frac{\left(\dfrac{dp/dx}{M^2} - 1\right)\beta \sinh \beta\eta \left(2\pi\varepsilon \cos(2\pi x)\right)}{\cosh^2 \beta\eta \left(1 - \dfrac{\beta^2}{\alpha^2}\right)}$$

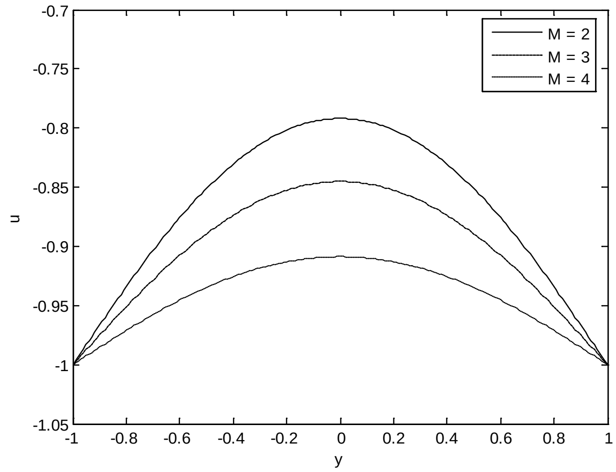

Fig. 2.1. variation of M on u when ε=0.02, x=0.5, S=0.75, p=1, M=2, 3, 4.

4 Results and Discussion

This section provides the behavior of various parameters involved in the expressions of amplitude of the peristaltic wave (ε), couple stress parameter(S), Velocity profile of the fluid (u), and the pressure gradient (dp/dx). Particularly, the variations of coefficient of couple stress(η), volume flow rate (q), Hartmann number (M), fluid pressure (p) are tested.

The behavior of parameters involved in Hartmann number is shows in Fig. 2.1. Fig.2.1 depicts that the Hartmann number (M) increases as velocity profile of the fluid (u)decreases.The behavior of parameters involved in couple stress parameter is shows in Fig. 2.2 when ε=0.02, dp/dx = 1, M = 3. It depicts that the couple stress parameter (S) increases as velocity profile of the fluid (u) decreases. The behavior of pressure gradient (dp/dx) on velocity profile u is shows in Fig. 2.3 when ε=0.02, S=0.75, M=3and dp/dx *=0.5, 1, 1.5*.Fig.2.3 depicts that the pressure gradient (dp/dx) increases as velocity profile of the fluid (u) increases. The behavior of couple stress parameter is shows in Fig. 3.1 on dp/dx when ε=0.1, M=1, S=0.5, 0.6, 0.75. Fig.3.1 shows that couple stress parameter (S) increases as pressure gradient (dp/dx) decreases. The behavior of parameters amplitude of the peristaltic wave (ε) parameter is shows in Fig. 3.2. For the fixed values of ε on dp/dx when S=0.75, M=1, ε=0.1, 0.2, 0.3. It showedthat the amplitude of the peristaltic wave (ε) increases as pressure gradient (dp/dx) increases. Fig.3.3 depicts that the Hartmann number (M) increases as pressure gradient (dp/dx) increases.

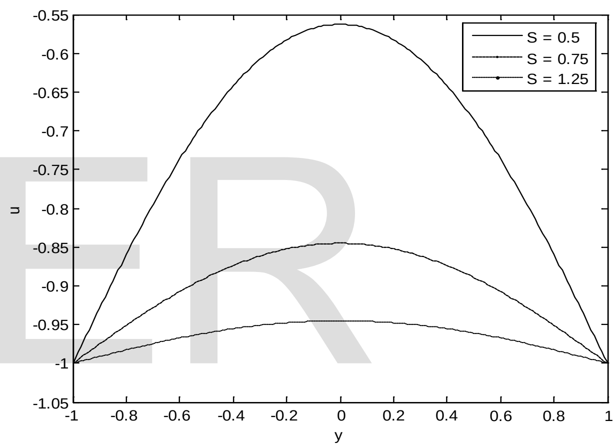

Fig. 2.2. variation of S on u when ε=0.02, x=0.5, p=1, M=3, S=0.5, 0.75, 1.25.

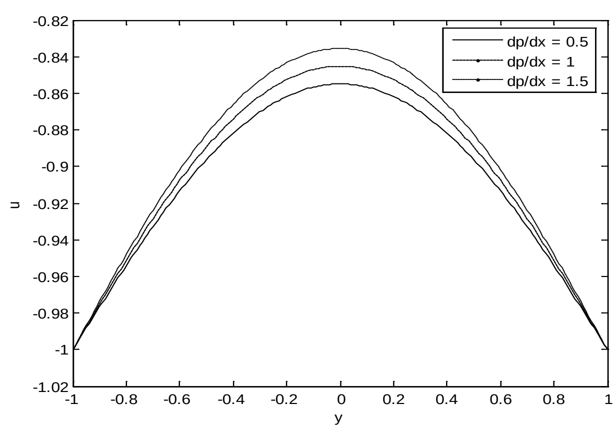

Fig. 2.3. variation of dp/dx on u when ε=0.02, x=0.5, s=0.75, p=1, M=3, *dp/dx=0.5, 1, 1.5.*

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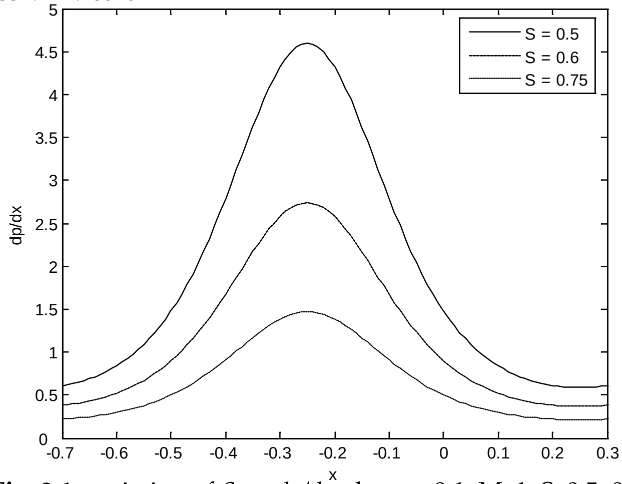

Fig. 3.1. variation of *S* on *dp/dx*when ε =0.1, M=1, S=0.5, 0.6, 0.75.

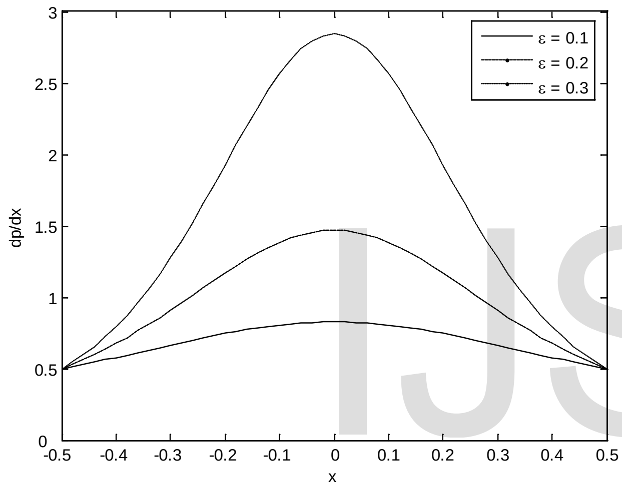

Fig. 3.2. variation of ε on *dp/dx* when S=0.75, M=1, ε =0.1, 0.2, 0.3.

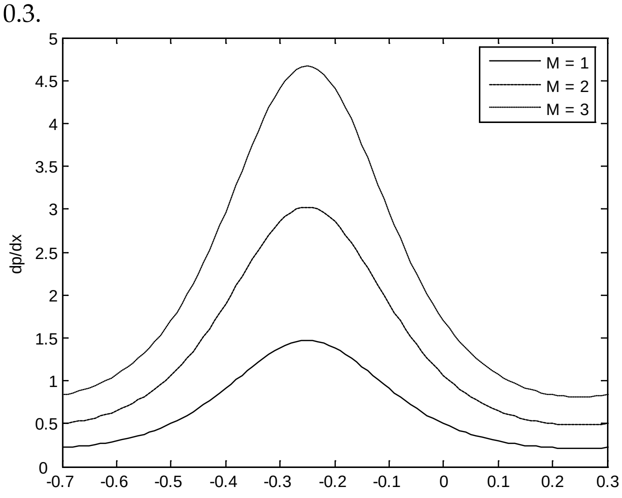

Fig. 3.3. variation of *M* on *dp/dx* when ε =0.1, S=0.75, M=1, 2, 3.

5 Conclusions

The influence of couple stress fluid on the MHD peristaltic flow through suspended particles in two dimensional flexible channels with analytical method of solution has been analyzed. The problem has been solved under long wave length and low Reynolds number approximation. The results calculated for velocity profile and pressure gradient. It is observed that u decreases with increase in M and S. The *dp/dx*has an opposite results compared with u for M.

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